Disagreement and the Option Stock Volume Ratio*

David C. Yang

December 2019

Abstract

This paper proposes a belief disagreement model to explain the empirical finding that the ratio of options-trading-volume-to-stock-trading-volume (O/S) negatively predicts cross-sectional stock returns (Johnson and So, 2012). Prior explanations focus on an asymmetric information model, where informed traders use options to express negative views. My belief disagreement model explains prior empirical findings and can better explain other empirical facts about O/S, including: (1) O/S is correlated with proxies for disagreement; (2) both put and call volume forecast negative returns in the cross section; (3) O/S predicts cross-sectional returns over a year later; (4) interaction effects with proxies for disagreement.

Keywords: option volume, stock volume, belief disagreement, cross section of expected returns

JEL Classifications: G12, G13

*Contact: david.yang@uci.edu, University of California, Irvine, Merage School of Business, 4291 Pereira Dr., Irvine, CA 92697. I thank John Campbell, Robin Greenwood, Charles Nathanson, Fan Zhang, and brownbag participants at Harvard University for helpful discussions and comments. The NSF Graduate Research Fellowship provided financial support for this research.

1 Introduction

What explains the relationship between trading volume in an options market and trading volume in its underlying market? Prices between the two markets are held together by theories such as Black and Scholes (1973) and Merton (1973). However, those theories are silent about the relationship of trading volume between the two markets. There is a literature that focuses on asymmetric information as an explanation for trading volume in the two markets. These papers emphasize that traders with private information transact in the options market to take advantage of the embedded leverage in options or to alleviate short-sale constraints (Black, 1975; Easley, O'Hara, and Srinivas, 1998). In contrast, in this paper, I build on the belief disagreement literature from behavioral finance to propose a disagreement-based model of the trade in an options market and its underlying market. I show how this model helps explain the negative correlation between O/S and the underlying asset's returns in the cross section. I also document other related empirical findings and show how the belief disagreement model explains those findings better than the asymmetric information model.

My empirical setting focuses on individual stocks and their options. To measure the relative behavior of trading volume in the options market and in the stock market, I focus on the ratio of options trading volume to stock trading volume, which Roll, Schwartz, and Subrahmanyam (2010) introduce and label as O/S. They discuss the possibility that O/S can be driven by asymmetric information or by belief disagreement. However, they focus on the time-series properties and determinants of O/S as opposed to its relationship with returns.

Johnson and So (2012) show that O/S is a negative predictor of cross-sectional stock returns: stocks with high O/S underperform stocks with low O/S. Extending the approach of Easley, O'Hara, and Srinivas (1998), they build a model of asymmetric information to explain their findings. In their model, short-sale constraints prevent investors from expressing negative bets in the stock market. They assume that the implied cost of shorting in the options market is lower because options market makers can more easily short stocks than investors. As a result, in their model, O/S is a predictor of negative private information and hence forecasts negative returns in the cross section. While their model has many rational components, Johnson and So (2012) note that their model requires that the investors in the underlying stock be somewhat irrational, since they do not react to the information embedded in options volume.

In contrast, consider a belief disagreement model, which also predicts a similar negative relationship between O/S and future returns. In this model, investors also face short-sale constraints. However, the fundamental reason that investors trade is because they disagree about the value of an asset, not because one particular set of investors knows more about the value of the asset as in the asymmetric information model. While investors can trade derivatives, the derivatives are not costless and hence do not fully offset the short-sale constraints. In this setting, the stock price becomes over-valued because some of the pessimists are shut out of the market (Miller, 1977). Furthermore, in this model, variation in either belief heterogeneity, risk tolerance, or supply of the underlying asset induces a negative correlation between the quantity of derivatives traded and expected returns, which corresponds to O/S and cross sectional returns in the empirical application. For example, an increase in the belief heterogeneity increases the price of the underlying asset due to the Miller (1977) effect. It also increases derivatives trade because the optimists and pessimists now have more extreme beliefs. Hence, it induces a negative correlation between derivatives trade and returns.

This belief disagreement model also produces several other predictions, which I document empirically in this paper and contrast with the predictions of asymmetric information model: First, the ratio of options trading volume to stock trading volume O/S is positively correlated with proxies for disagreement. Following Diether, Malloy, and Scherbina (2002), I use the dispersion in analyst forecasts as a proxy for the amount of belief heterogeneity. I verify that O/S is positively correlated with analyst forecast dispersion. In contrast, I find that O/S is negatively correlated with the bid-ask spread, which is traditionally thought of as a proxy for asymmetric information. This negative correlation is the opposite of the prediction in the asymmetric information model.

Second, the belief disagreement model can more easily explain the relationship between put volume and call volume and future cross-sectional returns. O/S is the total options trading volume divided by stock trading volume, and we can decompose O/S into a put-volume-version $(O/S)^{put}$ (i.e. put volume divided by stock volume) and a call-volume-version of $(O/S)^{call}$ (i.e. call volume divided by stock volume). In my belief disagreement model, higher disagreement increases both put volume and call volume and hence forecasts negative returns, which is what I document in the data.

In contrast, the asymmetric information model has potential empirical and theoretical shortcomings with respect to this fact. If the asymmetric information model assumes that investors are long-only, then put option volume should forecast negative returns and call option volume should forecast positive returns. However, this asymmetric information model prediction at odds with the data, which finds that both put option volume and call option volume forecast negative returns in the cross section. One could attempt to reconcile this empirical problem with the model by instead assuming that investors can also sell options. However, this assumption then creates a tension in the asymmetric information model: an investor who can sell options is unlikely to have a significant short-sale constraint, since both involve posting collateral and potentially unlimited losses. Without the short-sale constraint, the asymmetric information model no longer predicts that O/S forecasts negative returns. By comparison, the belief disagreement model is consistent with both put and call option volume forecasting negative returns, while still retaining the long-only constraint.

Third, I document empirically that O/S predicts cross-sectional expected returns over a year into the future. This empirical finding is at odds with the asymmetric information model. In the asymmetric information model, the investor with private information expresses his belief through the options market if it is negative news. This model also requires that the underlying stock market not react to trade in the options market. While this assumption is plausible over short time spans, the ability of O/S to predict cross-sectional returns 12+ month into the future is at odds with this assumption of the asymmetric information model. In fact, papers in the asymmetric information literature on options volume often focus on relatively short time spans. For example, Easley, O'Hara, and Srinivas (1998) focus on 30 minute prediction window and, using non-public data, Pan and Poteshman (2006) find predictability on the order of a few weeks. In contrast, in my disagreement model, the derivatives market and the underlying market are in equilibrium. No investor has private information that the other investors are trying to infer. Rather, investors simply agree to disagree with each other.

This long-term predictability is related to, but distinct from, Johnson and So (2012)'s findings. Their analysis uses data at a weekly frequency and finds that the cross-sectional predictability loses statistical significance at four to six weeks. My analysis uses data at a monthly frequency and finds statistical significance at longer time spans. Statistically, my result shows that the lagged values of O/S contain information beyond the latest value of O/S. One possibility is that factors outside my model also affect O/S, which makes the empirical O/S a noisy measure of the theoretical O/S. This noise creates downward bias in the magnitude of the effect and increases the standard errors. If these other factors are random shocks, then aggregating helps dilute the noise and improve the link between the empirical version of O/S and its theoretical counterpart.

Finally, I find strong interaction effects between O/S and analyst forecast dispersion, a proxy for investor disagreement (Diether, Malloy, and Scherbina, 2002). In general, O/S is a negative forecaster of returns in the cross section. However, when analyst forecast dispersion is high, O/S is an even stronger negative forecaster of returns in the cross section. My model suggests there are two competing effects here. In the model, variation in risk tolerance or in quantity of the underlying stock creates a negative correlation between O/S and expected returns. When belief heterogeneity is higher, the cross derivative implies that these factors have a weaker impact on price for each unit change. However, if higher belief heterogeneity is correlated with more variation in these factors, then the interaction effect goes the opposite direction. The empirics suggest that the latter effect is stronger.

The empirical part of my paper connects directly with papers on the option stock volume ratio, O/S. Roll, Schwartz, and Subrahmanyam (2010) introduce this measure and discuss some of its basic properties. Johnson and So (2012) document the relationship between O/S and cross-sectional stock returns. My paper proposes an alternate explanation based on disagreement and explains how the belief disagreement model helps us better understand other empirical facts about O/S. Ge, Lin, and Pearson (2016) use data on signed options volume to show that the forecasting power of O/S is due to embedded leverage and not due to alleviating short sale constraints. They briefly consider the relationship between O/S and disagreement, but reject it because it does not affect the predictability of O/S at short horizons. However, they note their analysis focuses on weekly frequencies and does not capture the potential effects of slow moving disagreement, which I study here. Johnson and So (2018) argue that the absolute changes in O/S measures the amount of information asymmetry. My paper is also related to other papers on options volume, such as Pan and Poteshman (2006), which uses proprietary data to compute the buyer-initiated put-call volume ratio and analyze its relationship with returns.

The theoretical part of my paper connects to the behavioral finance literature on belief disagreement as the reason for trading of financial assets (Harris and Raviv, 1993; Kandel and Pearson, 1995; Hong and Stein, 2007). My model builds on Miller (1977) and Chen, Hong, and Stein (2002) and extends it to allow for derivatives trade. Buraschi and Jiltsov (2006) construct a general equilibrium heterogeneous belief model to jointly explain volume and price in the options market, with a focus on the dynamics of the volatility smile. In contrast, I focus on the relationship between options trade and expected returns in the underlying stock. Belief disagreement models contrast with the literature on modeling trade in financial markets as the interaction between privately informed investors and noise/liquidity traders (Grossman and Stiglitz, 1980; Kyle, 1985). As emphasized by Hong and Stein (2007), disagreement models can better explain the large trading volumes we observe in financial markets. In asymmetric information models, trade requires liquidity shocks since uninformed but rational investors will infer the private information from market interactions with informed investors.

Section 2 describes the belief disagreement model and derives its implications. Section 3 describes my data sources, defines the key explanatory variables, and discusses summary statistics. Section 4 discusses the main empirical results. Section 5 concludes.

2 Model of Belief Disagreement and Derivatives Trade

My model builds on the belief disagreement models of Miller (1977) and Chen, Hong, and Stein (2002) and allows for derivative securities. The model has two time periods $t \in \{0, 1\}$. There are four types of assets: the risk-free asset, the risky asset ("underlying stock"), and two derivatives on the risky asset (a "positive bet" derivative and a "negative bet" derivative). I normalize the net risk-free rate to 0, by assuming that it has elastic supply at that rate. At time 1, each unit of the underlying stock pays a terminal dividend distributed $N(\mu, 1)$. The underlying stock has a total supply of Q_x. For tractability, I assume that both derivative securities are a leveraged payoff of the underlying stock. The "positive bet" derivative security pays n times the terminal dividend of the underlying stock, at time 1. The "negative bet" derivative security pays the opposite, i.e. $-n$ times the terminal dividend of the underlying stock, at time 1. I assume these are separate securities to allow investors with shorting constraints to make negative bets. The "positive bet" derivative and "negative bet" derivative are meant to be analogous to a call option and a put option, respectively. However, because the non-linearity of options creates tractability problems, I instead model derivatives securities that payoff a linear multiple ($\pm n$) of the underlying stock. In terms of quantities, since derivatives are contracts between agents inside the model, there are no exogenous constraints on the supply of derivatives.

There are two types of agents in the model: derivatives market makers and long-only buyers, each with a unit mass. The derivatives market makers exist only to create the two derivative securities. To prevent them from making other bets in the market, I assume that the derivatives market makers have infinite risk aversion and strive simply to minimize volatility of their wealth. For a cost $n \cdot c > 0$, the derivatives market maker can create a pair of "positive bet" derivatives and "negative bet" derivatives. The market makers assign the statutory incidence of the cost to the "negative bet" derivative. Because derivatives market makers create the derivatives in pairs, they are never exposed to any risk, satisfying their infinite risk aversion.

As their name suggests, long-only buyers can buy the underlying stock or either of the derivatives, but cannot short sell. There are a continuum of long-only buyers with heterogeneous beliefs about the mean payoff of the underlying stock. Long-only buyer i believes that the underlying stock will pay an uncertain terminal dividend with distribution $N(i, 1)$. For simplicity, I assume that they only disagree about the mean of the payoff and not the standard deviation. Let $i \sim Uniform[\mu - h, \mu + h]$ so the average long-only buyer has the correct belief about the mean payoff μ and the total heterogeneity is $2h$. The long-only buyer has constant relative risk aversion (CARA) utility with risk-tolerance τ over next period's wealth.

Let p_x, p_y, p_z denote the prices of the underlying stock, the "positive bet" derivative, and the "negative bet" derivative, respectively. Given the linearity of the setup, $p_y = n \cdot p_x$ and $p_z = -n \cdot p_x + n \cdot c$, since the derivatives have payoff of $\pm n$ times the underlying asset. In a more generalized (but less analytically tractable) model, investors could trade call options and put options instead of a "positive bet" derivative and a "negative bet" derivative. In this alternate setup, the price of the call option and put options would also be closely linked to the price of the underlying stock; however, the relationship would be non-linear as in most option pricing models.

For long-only buyer i, let x_i denote her demand for the underlying stock, y_i denote her demand for the "positive bet" derivative, and z_i denote her demand for the "negative bet" derivative. Because long-only buyers cannot short, I assume that $x_i \geq 0$, $y_i \geq 0$, and $z_i \geq 0$ for all i. In equilibrium, the long-only buyers will separate into three groups, $i \in [p_x, \mu + h]$, $i \in [p_x - c, p_x]$, $i \in [\mu - h, p_x]$.

Group $i \in [p_x, \mu + h]$ has an optimistic valuation of the underlying stock and has positive demand for the underlying stock and the "positive bet" derivative. In particular, their demand is $x_i + ny_i = \tau(i - p_x)$. The investors have indeterminate demand between the underlying stock and the "positive bet" derivative because they do not have borrowing constraints. In a more complicated model with borrowing constraints, the most optimistic investors would purchase the "positive bet" derivative for its leverage and the medium optimistic investors would purchase the underlying stock.

Group $i \in [p_x - c, p_x]$ has a moderately pessimistic valuation of the underlying stock and will only hold the risk-free asset. Despite their moderate pessimism, they cannot express their view in the market because the cost of creating a derivative exceeds their modest pessimism.

Group $i \in [\mu - h, p_x - c]$ has a strong pessimistic view of the underlying stock and expresses this negative view by purchasing the "negative bet" derivative. In particular, their demand is $z_i = \frac{\tau((p_x - c) - i)}{n}$ units of the "negative bet" derivative.

Setting supply equal to demand in the underlying stock market and the derivatives market, we get two market clearing conditions:

$$Q_x = \int_{p_x}^{\mu+h} \frac{x_i}{2h} di \tag{1}$$

$$\int_{\mu-h}^{p_x-c} \frac{z_i}{2h} di \quad = \quad \int_{p_x}^{\mu+h} \frac{y_i}{2h} di \tag{2}$$

As a benchmark, consider the case where $c = 0$. In this case, it is costless to create derivatives, which effectively removes the constraint that buyers must be long-only. In this situation, we have:

$$p_x^{*,bmark} = \mu - \frac{Q_x}{\tau} \tag{3}$$

That is, the price of the underlying stock is its true mean payoff less a risk premium. As the risk-tolerance τ approaches infinity, the risk-premium approaches zero and the price of the underlying stock approaches μ.

Next, I consider the case with general values of c. In this situation, the equilibrium price of the underlying stock is:

$$p_x^* = \mu - \frac{Q_x}{\tau} \cdot \frac{2h}{2h-c} + \frac{c}{2} \tag{4}$$

As a regularity condition, we require that:

$$p_x^{*,bmark} - c > \mu - h \tag{5}$$

which is equivalent to $h > c + \frac{Q_x}{\tau}$. Intuitively, we need enough belief heterogeneity so the shorting constraint binds.

I define the total quantity of derivatives sold in equilibrium as:

$$Q_z^* \quad := \quad \int_{\mu-h}^{p_x^*-c} \frac{z_i}{2h} di \tag{6}$$

Whereas the total supply of the underlying stock Q_x is exogenously fixed, Q_z^* is an endogenous variable since derivatives are synthesized by the market maker. Using this definition, I let Q_z^*/Q_x be the ratio of the quantity of derivatives to the quantity of the underlying stock.

Proposition 1. *The equilibrium price of the underlying stock p_x^* exceeds its benchmark price $p_x^{*,bmark}$.*

This proposition is analogous to the Miller (1977) result. Algebraically, it follows directly from computing the difference $p_x^* - p_x^{*,bmark}$ and imposing the regularity condition in Equation

5. Intuitively, the long-only buyers in the group $i \in [p_x^* - c, p_x^*]$ have a negative view of the asset. However, because they cannot short and because of the positive cost of creating the "negative bet" derivative, they are shut out of the market. As a result, p_x^* is higher than the benchmark price $p_x^{*,bmark}$.

Proposition 2. *When c is higher, the quantity of derivatives sold Q_z^* decreases.*

This proposition follows from taking the derivative and applying the regularity condition. Intuitively, when the cost of creating a derivative is higher, it is less attractive to create derivatives.

Proposition 3. *As belief heterogeneity h increases, the quantity of derivatives sold Q_z^* increases. Since the supply Q_x is held constant, Q_z^*/Q_x (the ratio of the quantity of derivatives sold to the quantity of stock) also decreases.*

The total quantity of "negative bet" derivatives sold by the market maker is Q_z^* (which equals the total quantity of "positive bet" derivatives sold by the market maker). This proposition follows from substituting in p_x^* and applying the regularity condition in Equation 5. Intuitively, as the belief heterogeneity increases, the optimists and the pessimists have more divergent beliefs and hence are more interested in purchasing the derivative securities.

Proposition 4. *As belief heterogeneity h increases, the equilibrium price of the underlying stock p_x^* is increases.*

This proposition comes directly from taking the first derivative of p_x^* with respect to h. As shown in Proposition 3, as belief heterogeneity increases, market makers create and sell more derivatives. However, there is still the original supply of Q_x of the underlying stock that the long-only buyers must own in equilibrium. The rise in the equilibrium price of the underlying stock p_x^* is necessary to clear that market.

Proposition 3 and Proposition 4 imply the following corollary:

Corollary 1. *Variation in belief heterogeneity h induces a positive correlation between the quantity of derivatives issued and the price of the underlying stock p_x^*—and hence a negative correlation between the quantity of derivatives sold Q_z^* and the expected return of the underlying stock $\mu - p_x^*$. Similarly, variation in h creates a negative correlation between Q_z^*/Q_x*

(the ratio of the quantity of derivatives sold to the quantity of stock) and $\mu - p_x^*$ (the expected return of the underlying stock).

Next, I derive comparative statics with respect to the risk tolerance τ and with respect to the supply of the underlying stock Q_x. In the model, changes in the Q_x are just changes in the supply of the underlying stock. However, we can also think of changes in supply Q_x as coming from changing investor sentiment, as in Chen, Hong, and Stein (2002).

Proposition 5. *As risk tolerance τ increases, p_x^* increases and the quantity of derivatives sold Q_z^* also increases. Since the supply Q_x is held constant, Q_z^*/Q_x (the ratio of the quantity of derivatives sold to the quantity of stock) also decreases.*

For both outcome variables (p_x^* and the quantity of derivatives issued), this proposition follows from taking the first derivative with respect to τ and using the regularity condition in Equation 5. Intuitively, as risk tolerance rises, the risk premium decreases, which increases p_x^*. Also, as risk tolerance rises, long-only buyers become more aggressive in their derivative bets on the positive and negative side, which increases the quantity of derivatives issued.

Proposition 6. *As the supply of the underlying stock Q_x increases, p_x^* decreases. Similarly, as the supply Q_x increases, the quantity of derivatives sold Q_z^* decreases and Q_z^*/Q_x (the ratio of the quantity of derivatives issued to the quantity of stock) also decreases.*

Intuitively, as the supply of the underlying stock increases, its price falls as there is now more supply for the buyers to absorb. The quantity of derivatives Q_z^* decreases because more of the optimists in the group $i \in [p_x^*, \mu + h]$ can simply now buy the underlying stock instead of the "positive bet" derivative. Since the quantity of derivatives Q_z^* decreases with an increase in Q_x, the ratio of the quantity of derivatives to the quantity of the underlying stock (i.e. Q_z^*/Q_x) also decreases with an increase in Q_x.

Proposition 5 and Proposition 6 imply the following two corollaries:

Corollary 2. *Variation in the risk tolerance τ or variation in the supply of the underlying stock Q_x induces a positive correlation between the equilibrium quantity of derivatives sold Q_z^* and the price of the underlying stock. This implies a negative correlation between the quantity of derivatives traded and the expected return of the underlying stock $\mu - p_x^*$. Similarly,* *variation in τ or Q_x creates negative correlation between Q_z^*/Q_x (the ratio of the quantity of derivatives issued to the quantity of stock) and $\mu - p_x^*$ (the expected return of the underlying stock). Because this negative correlation comes from variation in τ and Q_x, this negative correlation persists even if we hold belief heterogeneity constant.*

Corollary 3. *Belief heterogeneity h and Q_z^*/Q_x (the ratio of the quantity of derivatives issued to the quantity of stock) are correlated. However, their correlation will not be 1 since variation in risk tolerance τ or variation in the supply of the underlying stock Q_x also drives Q_z^*/Q_x.*

In the empirics, I focus on the negative correlation between Q_z^*/Q_x (the ratio of the equilibrium quantity of derivatives sold to the quantity of stock) and $\mu - p_x^*$ (the expected return of the underlying stock), as discussed in Corollary 1 and Corollary 2. I do not focus on the unscaled quantity of derivatives issued since that quantity is not comparable across stocks with differing amounts of shares outstanding. I examine both the volume ratio (options trading volume divided by stock trading volume) and the level ratio (using levels as opposed to flows, i.e. open interest in the options divided by shares outstanding in the underlying stock). Because this model only has two periods, there is not a meaningful distinction between the two measures in the model. Finally, in the empirical application, $\mu - p_x^*$ is best thought of as the alpha after adjusting for risk factors.

I also examine how $\frac{\partial p_x^*}{\partial \tau}$ and $\frac{\partial p_x^*}{\partial Q_z}$ changes as h increases:

Proposition 7. *As h increases, $\frac{\partial p_x^*}{\partial \tau}$ declines (i.e. $\frac{\partial p_x^*}{\partial \tau \partial h} < 0$), but $\frac{\partial p_x^*}{\partial \tau}$ is still positive overall. Also, as h increases, $\frac{\partial p_x^*}{\partial Q_x}$ rises (i.e. $\frac{\partial p_x^*}{\partial Q_x \partial h} > 0$), but $\frac{\partial p_x^*}{\partial Q_x}$ is still negative overall.*

This proposition states that when h is larger, each unit increase in τ still increases p_x^*, but at a slower rate (and vice versa for changes in Q_x). This proposition however does not necessarily determine how the relationship between Q_z^*/Q_x and future returns $\mu - p_x^*$ varies with h. For example, if there is more τ variation when h is higher, then we may observe a stronger relationship between Q_z^*/Q_x and future returns $\mu - p_x^*$ for high h, despite the cross derivative.

3 Data, Variable Definitions, and Summary Statistics

The option-level data in this paper come from OptionMetrics' Ivy Database. This database covers all US exchange-listed options starting from January 1996. Because this paper focuses on cross-sectional variation, I use data on options on individual stocks, as opposed to data on options on stock indices. For each option, I pull the data on its volume, its type (i.e. put vs call), and its expiration date. The OptionMetrics data is at a daily frequency and I aggregate to construct monthly measures. I omit firm-month observations for which there is no options volume in the entire month, across all option types.

The stock-level data in this paper come from the Center for Research in Security Prices (CRSP), Standard and Poor's COMPUSTAT, Ken French's Data Library, and Thomson Reuters' Institutional Brokers Estimate System (I/B/E/S). From CRSP, I use the Monthly Stock File to obtain data on monthly holding period returns, volume, close prices, bid and ask prices, etc. From COMPUSTAT, I obtain balance sheet data to compute accounting book value. From I/B/E/S, I obtain data on individual analyst forecasts of future earnings per share.

The main explanatory variable in this paper is the option-stock volume ratio (O/S), as introduced by Roll, Schwartz, and Subrahmanyam (2010) and studied by Johnson and So (2012). While prior work has studied O/S at a weekly frequency, this paper studies it at a monthly frequency. As its name suggests, O/S is defined as the ratio of options volume to stock volume. For a given stock on a given day, there are many different types of options, e.g. put vs call, different strike prices, different expiration dates. To measure "options volume," we sum the volume over all the different types of options. Specifically, let k index all the different type of options. Then, let $OptionVolume_{i,t,k}$ denote the volume of an option of type k on a given stock i at time t. For example, on December 31, 2012, there were 3904 contracts traded for the put option on Apple, Inc. stock with an expiration date of February 16, 2013 and a strike price of \$600. I define O/S as:

$$(O/S)_{i,t} := \frac{\sum_k 100 \cdot OptionVolume_{i,t,k}}{StockVolume_{i,t}} \tag{7}$$

I multiply $OptionVolume_{i,t,k}$ by 100 because each options contract corresponds to 100 shares of the underlying stock. I also study some specialized versions of O/S defined over specific subsets of options. For example, O/S where options volume is summed only over put options or only over call options. In each of these instances (e.g. Table 5), I denote the subset used.

I also study $LevelO/S$, which defines O/S in terms of levels instead of flows/volumes:

$$(LevelO/S)_{i,t} := \frac{\sum_k 100 \cdot OptionOpenInterest_{i,t,k}}{StockSharesOutstanding_{i,t}} \tag{8}$$

I measure investor disagreement using the dispersion in analyst forecasts ($Disp$), as used in Diether, Malloy, and Scherbina (2002). From the I/B/E/S Unadjusted Detail file, I pull the individual analyst forecasts of earnings per share for the next fiscal year. As highlighted by Diether, Malloy, and Scherbina (2002), it is important to use the original forecasts, unadjusted for share splits, due to rounding issues in I/B/E/S. For stock i at time t, let $forecast_{i,t,j}$ denote the forecast by analyst j. Then, $Disp$ is defined as

$$Disp_{i,t} := \frac{\sigma(forecast_{i,t,j})}{|E[forecast_{i,t,j}]|} \tag{9}$$

For ease of reference, in Table 1, I list the key variables in this paper and their definitions. Table 2a lists the summary statistics for the full dataset. In the time series and the cross section, the main constraint is the OptionMetrics database. In terms of time span, my dataset covers 1996 to 2013 for 211 months of observations. In terms of the cross section, my dataset has roughly 1500 firms per cross section. Since my dataset omits stocks without any options, the average stock in my dataset has a market capitalization of roughly $6 billion.

Table 2b displays the mean of selected variables, across the quintiles of O/S. Stocks with higher O/S are have larger market capitalizations, lower book to market ratios, and are more liquid (higher turnover, lower spread). Stocks with higher O/S also have higher dispersion in analyst forecasts and higher stock return volatility.

4 Main Empirical Results

In this section, I compare and contrast the predictions of the model in Section 2 with the data. A key quantity in my model is Q_z^*/Q_x (the ratio of the quantity of derivatives issued to the quantity of stock). My model is a two period model, so taken literally, the model does not distinguish between volume and shares outstanding. As a result, in the empirics, I examine both the flow version O/S and the level version $LevelO/S$. Since prior work in Roll, Schwartz, and Subrahmanyam (2010) and Johnson and So (2012) focus on O/S, I take that as the main baseline and examine $LevelO/S$ as an alternate definition. Where appropriate, I also discuss how the empirical results help us distinguish between the belief disagreement model and the asymmetric information model in Johnson and So (2012).

4.1 Correlations with O/S

Corollary 3 predicts that belief heterogeneity should be correlated with O/S and $LevelO/S$. Following Diether, Malloy, and Scherbina (2002), I proxy belief heterogeneity using the dispersion in analyst forecasts *Disp*. The correlation in Table 3 verifies this prediction. Both O/S and $LevelO/S$ have a positive and statistically significant correlation with *Disp*. In the model, this correlation comes from the fact that changes in belief heterogeneity cause changes in the quantity of derivatives traded. Furthermore, while the correlation is statistically significant, it is much less than 1. This is also consistent with Corollary 3. In my model, changes in risk tolerance and changes in the total supply of the underlying stock also affect the quantity of derivatives. As noted in Section 2, changes in the supply of the underlying stock can be interpreted as literal changes in supply or also changes in investor sentiment. Because these other factors also affect the quantity of derivatives, it limits the strength of the correlation between *Disp* and O/S (or $LevelO/S$).

Table 3 also displays correlations that are contrary to the asymmetric information model of options trade. In the table, we see that higher bid-ask spreads in the underlying stock *Spread* are associated with lower values of O/S and $LevelO/S$. However, bid-ask spreads are often associated with asymmetric information (Glosten and Milgrom, 1985). Hence, the data suggest that higher O/S is associated with lower asymmetric information, which is contrary to the asymmetric information model.

4.2 Long-Short Portfolios

Table 4 displays the alphas of long-short portfolios. I form the long-short portfolios by sorting stocks into quintiles using the previous month's O/S (Columns (1) - (3)) or $LevelO/S$ (Columns (4) - (6)). Let $R_{Q5,t+1} - R_{Q1,t+1}$ denote the equal-weighted return spread between the highest quintile and the lowest quintile. I then run the time series regression

$$R_{Q5,t+1} - R_{Q1,t+1} = \alpha + \beta' \cdot f_{t+1} + \epsilon_{t+1} \tag{10}$$

where f_{t+1} is a vector of return factors. I initially control for the Carhart (1997) four-factor model, which includes the market factor $Rm - Rf$, the size factor SMB, the value factor HML, and the momentum factor UMD (Column (1) and (4)). Across the different specifications, there is a significant negative loading on the HML factor. This loading is consistent with the summary statistics in Table 2, which shows that the book-to-market ratio increases with the quintile of O/S. I then add the short-term reversal factor $STRev$ and the long-term reversal factor $LTRev$ (Columns (2) and (5)). Finally, I add the Pastor and Stambaugh (2003) liquidity factor LIQ and the Frazzini and Pedersen (2014) betting against beta factor BAB (Columns (3) and (6)). Since Table 2 shows that O/S is correlated with liquidity in the underlying stock (higher turnover, lower spreads), I control for LIQ to check that the liquidity factor does not drive the O/S return spread. I control for BAB because the long-short portfolio has a low loading on the market factor, so it is important to verify that BAB does not drive the O/S return spread.

Across the various specifications, several patterns emerge. First, the alphas are all consistently negative with a coefficient size of about -1% per month and a t-statistic in excess of 5. Harvey, Liu, and Zhu (2016) stress that t-statistics for cross-sectional expected return tests should exceed 3 in absolute value, due to data mining concerns. All the specifications here pass that test. Second, we see that the additional controls beyond the Carhart (1997) four-factor model have minimal loadings. For this reason, we use the Carhart (1997) four-factor model as the basic set of controls in later analyses. The findings in this table are consistent with the prior work of Johnson and So (2012). They focus on weekly returns, whereas I focus on monthly returns. Another minor difference is that this table shows the predictability of a new variable $LevelO/S$, which they do not consider.

Table 5 examines the effect of forming O/S with different subsamples. Recall that O/S is defined as (Total Trading Volume of Across All Options)/(Trading Volume of the Underlying Stock). However, we can also define alternate versions of O/S only using a subsample of options. For example, we could define $(O/S)^{put}$ where the numerator is instead Total Trading Volume of Put Options. Table 5 examines the results of forming alternate versions of O/S using different subsamples. Columns (1) defines O/S using only put option volume. Column (2) uses only call option volume. Column (3) uses only near-dated options, which I define as options expiring within 30 days. Column (4) uses only far-dated options, which I define as options expiring after 30 days. Since Table 4 showed that return factors beyond the Carhart (1997) four-factor model do not have significant loadings, I use the four-factor model as a control. In all subsamples, we continue to see statistically significant negative alphas with t-statistics exceeding 4.5 in absolute value.

Having described the basic features of Table 4 and Table 5, I now relate it to the model in Section 2. The negative alphas are consistent with the predictions from Corollary 1 and Corollary 2. In my belief disagreement model, variation in belief heterogeneity, risk tolerance, or supply shocks, all induce a negative correlation between O/S and returns. Furthermore, in the model, trade in both the "positive bet" derivative (the model's analogy of call options) and the "negative bet" derivative (the model's analogy of put options) is associated with negative returns. This is consistent with the empirical evidence that $(O/S)^{put}$ and $(O/S)^{call}$ forecast negative returns in the cross section.

In contrast, an asymmetric information model of options trade has empirical and theoretical tensions regarding the relationship between put/call volume and returns. If the informed traders in the asymmetric information model are long-only, then put option volume should forecast negative returns and call option volume should forecast positive returns. However, Table 5 shows that this prediction is counterfactual.

One can attempt to fix the asymmetric information model by allowing informed traders to both buy and sell options. This fix allows the informed trader to act on negative news by selling call options and hence potentially predicting that $(O/S)^{call}$ forecasts negative returns. While this model resolves the empirical issue, it creates a theoretical tension because it is unclear if an investor who can sell options would still have a significant short-sale constraint. In the asymmetric information model, the short-sale constraint is the key reason that an informed trader with negative information chooses to trade in the options market, instead of the stock market. Without the short-sale constraint, the asymmetric information model does not predict that O/S forecasts negative returns in the cross section. In contrast, my disagreement model can explain the reason both put and call option volume forecast negative returns, while still retaining the long-only constraint.

Table 6 displays the results of a Fama and MacBeth (1973) regression on the cross section of individual stock returns. This analysis is similar to the portfolio sorting analysis from above, with some small differences. The Fama-MacBeth methodology allows us to control for individual firm characteristics, as opposed to covariances with return factors such as SMB or HML (Daniel and Titman, 1997). However, one drawback of the Fama-MacBeth methodology is that it is sensitive to outliers and regression misspecification. Hence, it is standard to analyze cross-sectional predictability using both portfolio sorts and Fama-MacBeth regressions, as I do here.

Table 6 runs the following Fama-MacBeth regression

$$R^e_{i,t+1} = b_{0,t} + \lambda \cdot (\text{Measure of } O/S)_{i,t} + b_1 \cdot X_{i,t} + \epsilon_{i,t+1} \tag{11}$$

The dependent variable is the excess monthly return of a stock $R^e_{i,t+1}$, which is its monthly return less the one month Treasury bill rate. For controls $X_{i,t}$, I use the firm characteristics of log market capitalization, log book to market ratio, the CAPM beta, returns over the past 12 months (excluding the most recent month), returns over the past month, and turnover.

Across the different columns, I use variants of O/S. Column (1) uses the quintile of O/S and Column (2) uses the quintile of $Level O/S$. In both of these regressions, the Fama-MacBeth coefficient is roughly $\lambda = -0.13$ with a t-statistic exceeding 3 in magnitude. The coefficient of -0.13 corresponds to a return difference between the highest quintile and low quintile of $-0.52\% = -0.13\% \times (5 - 1)$ per month, which is smaller than the portfolio sort alpha of roughly -1% in Table 4.

Column (3) uses O/S itself and Column (4) uses $Level O/S$ itself. These regressions differ from the regressions in Column (1) and Column (2) in that they allow the Fama-MacBeth regression to use variation in the dispersion of O/S or $LevelO/S$. Column (1) and Column (2) suppress such information because it uses quintiles of O/S or $LevelO/S$, which forces a constant dispersion over time. In Column (3), the coefficient on O/S is -1.7 (with $|t| > 3$). If we take the average value of O/S in the highest quintile and the lowest quintile, the regression in Column (3) predicts a high-low-quintile return spread of -0.32% per month. In Column (4), the coefficient on $LevelO/S$ is -0.40 (with $|t| > 3$), which predicts a high-low-quintile return spread of -0.48% per month.

4.3 Effect of Changing the Portfolio Formation Period

In the previous section, I sorted stocks into portfolios using the previous month's O/S quintiles. In this section, I examine the effect changing the portfolio formation period. Consider sorting stocks into portfolios using the O/S quintiles from j months ago. For example, $j = 1$ would correspond to the sorting methodology from the previous section. Let $R^{(j)}_{Q5,t+1} - R^{(j)}_{Q1,t+1}$ denote the equal-weighted return spread formed using O/S from j periods ago, i.e. $(O/S)_{i,t-j}$. I run the time-series regression

$$R^{(j)}_{Q5,t+1} - R^{(j)}_{Q1,t+1} = \alpha^{(j)} + \beta' \cdot f_{t+1} + \epsilon_{t+1} \tag{12}$$

For the return factors f_{t+1}, I use the Carhart (1997) four-factor model, since Table 4 shows that return factors beyond the four-factor model do not have significant loadings. While I examine the effects of changing the lag length of the portfolio formation, I keep the portfolio holding period to be one month, so the dependent variable is always the one month return spread between the quintiles. Figure 1 plots the alphas $\alpha^{(j)}$ against the lag length of j months. For both O/S and $LevelO/S$, there is significant persistence in the cross-sectional predictability. For both variables, using the O/S (or $LevelO/S$) value today can predict cross-sectional returns over a year into the future.

In contrast, Johnson and So (2012) only find predictability up to 4-6 weeks into the future. Empirically, the reason for the differing findings is the frequency of the data. They use data at a weekly frequency and I use data at a monthly frequency. The fact that I find longer term cross-sectional predictability suggests that the lagged values of O/S contain information beyond the latest value of O/S. In undisplayed results, I check that forecasting next week's returns using a lagged moving average O/S yields larger estimates and t-statistics than only using last week's O/S.

Measurement error is one possible reason that using monthly O/S has more statistical power than weekly O/S. In my model, variation in both belief heterogeneity and risk tolerance creates a negative relationship between the theoretical version of O/S and future returns. However, in the data, O/S can vary due to other factors. If these other factors are random shocks, then weekly O/S has more measurement error than monthly O/S (relative to the theoretical counterpart in my model). This measurement error creates a downward bias in the magnitude of the effect and also reduces the statistical significance.

An alternate possibility is that aggregating the dependent variable (returns) increases the statistical significance. I do not believe that is the underlying reason for the difference between my results and Johnson and So (2012), but I explain this possibility for completeness. In the time-series return predictability literature, long horizon return regressions on dividend yield have larger coefficients and stronger statistical significance than short horizon regressions. Campbell (2001) and Cochrane (2007) explain that this finding is because (1) dividend yields are persistent and (2) return shocks and dividend yield shocks are negatively correlated, due to the Campbell and Shiller (1988) decomposition. One would need to first adapt those theories to the portfolio sorting test methodology that cross-sectional asset pricing papers use. Furthermore, in the case of O/S and its quintiles, the regressor has much less persistence and there is not necessarily a clear reason for a negative relationship between shocks to returns and shocks to the regressor.

Panel (b) illustrates this result in a different way, using $LevelO/S$, which is a version of O/S not previously examined in the literature. $LevelO/S$ is the ratio of options open interest and stock shares outstanding (as opposed to options volume and stock volume, which is a flow variable). Since $LevelO/S$ is defined using a level as opposed to flow measure of quantity, it is less subject to weekly variation. Panel (b) shows that $LevelO/S$ has slightly larger coefficients and forecasts returns at a slightly longer time spans than O/S itself.

The results in Figure 1 can also help us distinguish between the asymmetric information model and the belief heterogeneity model. In the asymmetric information model, investors with private negative information express their negative beliefs in the options market due to short-sale constraints. However, in this model, it is important that the underlying stock price does not react to options trade. Over short time spans, this assumption seems reasonable. However, the empirical analysis in Figure 1 shows that O/S can predict cross-sectional returns over a year later, which creates empirical tension for the asymmetric information model.

In contrast, the belief heterogeneity model accommodates the long term predictability more easily. In the belief heterogeneity model, increases in belief heterogeneity drive optimists and pessimists to purchase more derivatives. At the same time, belief heterogeneity increases the underlying price due to short-sale constraints. In this model, the prices in the derivatives market and the underlying market are in equilibrium (though, as with all disagreement models, the equilibrium is because the investors agree to disagree). Furthermore, it seems plausible that disagreement in beliefs between investors in the market can persist over a year.

4.4 Interaction Effects with Analyst Forecast Dispersion

Here I examine the interaction effect between O/S and analyst forecast dispersion $Disp$. The dispersion in analyst forecasts is often used as a proxy for belief disagreement between investors. Diether, Malloy, and Scherbina (2002) show that stocks with higher analyst forecast dispersion have lower returns, which they argue is evidence supporting the belief disagreement model of Miller (1977). For stock i at time t, let $forecast_{i,t,j}$ denote the forecast by analyst j. Then, $Disp$ is defined as[1]

$$Disp_{i,t} := \frac{\sigma(forecast_{i,t,j})}{|E[forecast_{i,t,j}]|}$$

Since this analysis requires data on analyst forecasts, the dataset in this subsection is a subset of the dataset in the rest of this paper because there are stocks with options, but no analyst coverage. The dataset spans also 1996 to 2013, but has roughly 30% fewer firm-month

[1] Some papers also use an alternate definition of $Disp$ where the denominator is the unadjusted share price, instead of the mean analyst forecast of earnings per share. This alternate definition can help potentially avoid divide by zero problems when the earnings per share is very low. Though not displayed, I have also computed this alternate definition and found similar results.

observations ($N = 233,049$).

Table 7a shows the average excess monthly returns of a two way portfolio sort between the quintiles of O/S vs the quintiles of analyst forecast dispersion $Disp$. As we move down the table, the quintile of O/S increases. As we move the right of the table, $Disp$ increases. The bottom row of Table 7a displays the average monthly excess return of the high-low-quintile return spread of O/S, across the different quintiles of $Disp$. For example, if we fix $Qtile(Disp) = 1$, the average monthly excess return of $Qtile(O/S) = 5$ minus the average monthly excess return of $Qtile(O/S) = 1$ is -0.57% per month. Furthermore, we observe that the size of this return spread is increasing with $Qtile(Disp)$. That is, there is an interaction effect between O/S and $Disp$.

While the two way portfolio sort in Table 7a is a convenient non-parametric way to look at the pattern between O/S and $Disp$, one might wonder if the interaction effect persists after controlling for other variables. To assess whether the interaction effect survives the addition of controls, Table 7b uses Fama-MacBeth regressions on the cross section of individual stock returns. I run the regression

$$\begin{aligned} R^e_{i,t+1} &= b_{0,t} + \lambda_1 \cdot Qtile(O/S)_{i,t} + \lambda_2 \cdot Qtile(O/S)_{i,t} \times Qtile(Disp)_{i,t} \\ &\quad + \lambda_3 \cdot Qtile(Disp)_{i,t} + b_1 \cdot X_{i,t} + \epsilon_{i,t+1} \end{aligned} \tag{13}$$

The dependent variable is excess monthly return of a given stock i in a given month $t + 1$. The controls are the same as the controls in Table 6, i.e. log market capitalization, log book to market ratio, the CAPM beta, returns over the past 12 months (excluding the most recent month), returns over the past month, and turnover.

Because this table also examines an interaction effect, I cross-sectionally de-mean the controls and the quintiles. For the terms uninvolved in the interaction, this de-meaning does not affect the regression coefficient, since the Fama-MacBeth regression allows for a different intercept for each time period. However, it is important to cross-sectionally de-mean the variables $Qtile(O/S)$ and $Qtile(Disp)$. Otherwise, in the regression with the interaction term, the coefficient λ_1 on $Qtile(O/S)$ will correspond to the effect when $Qtile(Disp) = 0$, which is undefined as quintiles range from 1 to 5. By cross-sectionally de-meaning, the coefficient λ_1 on $Qtile(O/S)$ corresponds to the effect for a firm with an average value of $Qtile(Disp)$, which is 3.

Column (1) of this table is analogous to Column (1) of Table 6, but for the subset of data with analyst forecasts. I verify that the coefficients are essentially the same, confirming that this dataset is a reasonably representative sample of the dataset used in the rest of this paper. In Column (2), I additionally control for the quintile of *Disp*. It only marginally affects the coefficient on the quintile of O/S. Over this time interval, *Disp* is not statistically significant; however, I have verified that it has a negative statistically significant effect over a larger time span, confirming prior work using *Disp*. In Column (3), I add the interaction term $Qtile(O/S) \times Qtile(Disp)$. It is statistically significantly negative with $\lambda_2 = -0.04$ with $|t| > 3$. That is, higher O/S is associated with lower returns, but the relationship is even stronger in magnitude when there is higher analyst forecast dispersion.

In terms of the theory, this interaction effect appears to come from the fact that higher h is associated with more variation in the variables that drive O/S and expected returns. My model in Section 2 shows that variation in risk tolerance or in quantity of the underlying stock creates a negative correlation between O/S and expected returns. If the variation in the underlying variables is constant across quintiles of h, then the cross derivatives in Proposition 7 suggests that the return spread should fall with higher belief heterogeneity h. However, if there is more variation in the underlying variables when h is higher, then the return spread increases with h. The interaction effect shown in Table 7 suggests that the latter effect dominates. The latter effect also predicts that there should be more variation in O/S when h is higher. Table 2c suggests that is true, though the relationship is not perfectly monotonic.

Yet another possibility is that there are other dynamics between belief heterogeneity, O/S, and expected returns that my model in Section 2 does not capture. Nonetheless, the results in Table 7 suggest that there is a relationship between belief heterogeneity and the ability of O/S to forecast cross-sectional returns.

5 Conclusion

In this paper, I propose a belief disagreement model to explain the relative trading volume in an options market and its underlying market. Johnson and So (2012) find that the ratio of option trading volume to stock trading volume O/S negatively predicts stock returns in the cross section. They propose an asymmetric information model where investors with private negative information prefer to trade in the options market because they face shorting constraints in the underlying stock market. To prevent the stock market in their model from reacting immediately, Johnson and So (2012) assume that investors in the underlying stock are somewhat irrational and do not react to options volume.

I show how a belief disagreement model in the spirit of Miller (1977) and Chen, Hong, and Stein (2002) also predicts a negative relationship between O/S and the expected return. I also document the following empirical findings and explain how the belief disagreement model is more consistent with these empirical results than an asymmetric information model: (1) O/S is positively correlated with analyst forecast dispersion, a proxy of belief heterogeneity, but is negative correlated with the bid-ask spread, a proxy of asymmetric information; (2) Both put option volume and call option volume are negative predictors of future returns in the cross section; (3) O/S is a negative predictor of cross-sectional returns over one year into the future; (4) The negative relationship between O/S and cross-sectional returns is stronger when analyst forecast dispersion is larger.

References

[1] Black, F. (1975). Fact and fantasy in the use of options. Financial Analysts Journal, 31(4), 36-41.

[2] Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. The Journal of Political Economy, 81(3), 637-654.

[3] Buraschi, A., & Jiltsov, A. (2006). Model uncertainty and option markets with heterogeneous beliefs. The Journal of Finance, 61(6), 2841-2897.

[4] Carhart, M. M. (1997). On persistence in mutual fund performance. The Journal of Finance, 52(1), 57-82.

[5] Campbell, J. Y., & Shiller, R. J. (1988). The dividend-price ratio and expectations of future dividends and discount factors. Review of Financial Studies, 1(3), 195-228.

[6] Cochrane, J. H. (2008). The dog that did not bark: A defense of return predictability. Review of Financial Studies, 21(4), 1533-1575.

[7] Daniel, K., & Titman, S. (1997). Evidence on the characteristics of cross sectional variation in stock returns. The Journal of Finance, 52(1), 1-33.

[8] Diether, K. B., Malloy, C. J., & Scherbina, A. (2002). Differences of opinion and the cross section of stock returns. The Journal of Finance, 57(5), 2113-2141.

[9] Easley, D., O'Hara, M., & Srinivas, P. S. (1998). Option volume and stock prices: Evidence on where informed traders trade. The Journal of Finance, 53(2), 431-465.

[10] Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. Journal of Financial Economics, 33(1), 3-56.

[11] Fama, E. F., & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. The Journal of Political Economy, 81(3), 607-636.

[12] Frazzini, A., & Pedersen, L. H. (2014). Betting against beta. Journal of Financial Economics, 111(1), 1-25.

[13] Ge, L., Lin, T. C., & Pearson, N. D. (2016). Why does the option stock volume ratio predict stock returns? Journal of Financial Economics, 120(3), 601-622.

[14] Glosten, L. R., & Milgrom, P. R. (1985). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. Journal of Financial Economics, 14(1), 71-100.

[15] Grossman, S. J., & Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. The American Economic Review, 70(3), 393-408.

[16] Harris, M., & Raviv, A. (1993). Differences of opinion make a horse race. Review of Financial Studies, 6(3), 473-506.

[17] Harvey, C. R., Liu, Y., & Zhu, H. (2016). ... And the cross-section of expected returns. Review of Financial Studies, 29(1), 5-68.

[18] Hong, H., & Stein, J. C. (2007). Disagreement and the stock market. Journal of Economic perspectives, 21(2), 109-128.

[19] Johnson, T. L., & So, E. C. (2012). The option to stock volume ratio and future returns. Journal of Financial Economics, 106(2), 262-286.

[20] Johnson, T. L., & So, E. C. (2018). A simple multimarket measure of information asymmetry. Management Science, 64(3), 1055-1080.

[21] Kandel, E., & Pearson, N. D. (1995). Differential interpretation of public signals and trade in speculative markets. Journal of Political Economy, 103(4), 831-872.

[22] Kyle, A. S. (1985). Continuous auctions and insider trading. Econometrica, 53(6), 1315-1335.

[23] Merton, R. C. (1973). Theory of rational option pricing. The Bell Journal of Economics and Management Science, 4(1), 141-183.

[24] Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion. The Journal of Finance, 32(4), 1151-1168.

[25] Newey, W. K., & West, K. D. (1986). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica, 55(3), 703-708.

[26] Pan, J., & Poteshman, A. M. (2006). The information in option volume for future stock prices. Review of Financial Studies, 19(3), 871-908.

[27] Pastor, L., & Stambaugh, R. F. (2003). Liquidity risk and expected stock returns. Journal of Political Economy, 111(3), 642-685.

[28] Roll, R., Schwartz, E., & Subrahmanyam, A. (2010). O/S: The relative trading activity in options and stock. Journal of Financial Economics, 96(1), 1-17.

[29] Scholes, M., & Williams, J. (1977). Estimating betas from nonsynchronous data. Journal of Financial Economics, 5(3), 309-327.

A Proofs

Most of the proofs are computing the derivative and recognizing different forms of the regularity condition. The regularity condition in Equation 5 ensures that the shorting constraint binds. In the proofs, I apply several different forms of the regularity condition, which are all equivalent. I list them here:

$$\begin{aligned} p_x^{*,bmark} - c &> \mu - h \\ \Leftrightarrow h &> c + \frac{Q_x}{\tau} \\ \Leftrightarrow h\tau - c\tau - Q_x &> 0 \\ \Leftrightarrow 4h^2\tau - 4hQ_x - 4ch\tau &= 4h(h\tau - Q_x - c\tau) > 0 \\ \Leftrightarrow 2h - c &> h + \frac{Q_x}{\tau} > 0 \\ \Leftrightarrow c - 2h &< -h - \frac{Q_x}{\tau} < 0 \end{aligned}$$

It is also convenient to solve out Q_z^* in terms of the underlying parameters of the model. Combining Equation 4 and Equation 6 from the main text, we get:

$$Q_z^* = \frac{1}{4hn} \cdot \tau(\frac{c}{2} + h(\frac{2Q_x}{2h\tau - c\tau} - 1)) \tag{14}$$

Proof of Proposition 1: Applying Equation 3 and Equation 4, we get

$$\begin{aligned} p_x^* - p_x^{*,bmark} &= c(\frac{1}{2} - \frac{1}{2h - c}\frac{Q_x}{\tau}) \\ &= c(\frac{2h - c - 2\frac{Q_x}{\tau}}{2(2h - c)\tau}) \\ &> 0 \end{aligned}$$

The denominator is positive since $2h - c > 0$ by the regularity condition. The numerator is also positive since $2h - c > 2h - 2c > 2\frac{Q_x}{\tau}$, where the last inequality follows from the regularity condition.

Proof of Proposition 2: Using Equation 14 and taking the derivative with respect to the belief heterogeneity c, we get:

$$\frac{\partial Q_z^*}{\partial c} = \frac{-1}{8h(2h-c)^3}(c^2\tau + 4h^2\tau - 4hQ_x - 4ch\tau)(c^2\tau + 4h^2\tau + 4hQ_x - 4ch\tau)$$

By the regularity condition, $2h - c > 0$. Also $4h^2\tau - 4hQ_x - 4ch\tau > 0$, by the regularity condition. This implies that $4h^2\tau + 4hQ_x - 4ch\tau > 4h^2\tau - 4hQ_x - 4ch\tau > 0$. Therefore, the derivative is negative.

Proof of Proposition 3: Using Equation 14 and taking the derivative with respect to the belief heterogeneity h, we get:

$$\frac{\partial Q_z^*}{\partial h} = \frac{c^2\tau + 4h^2\tau - 4h(Q_x + c\tau)}{16h^2(2h-c)^3\tau^2} \cdot \frac{(c+2h)(c^2\tau + 4h^2\tau + 4h(Q_x - c\tau))}{n}$$

By the regularity condition, $2h - c > 0$. Next, $4h^2\tau - 4h(Q_x + c\tau) = 4h(h\tau - Q_x - c\tau) > 0$ by the regularity condition. Finally, $4h^2\tau + 4h(Q_x - c\tau) = 4h(h\tau + Q_x - c\tau) > 4h(h\tau - Q_x - c\tau) > 0$ where the last step follows from the regularity condition. Therefore, $\frac{\partial Q_z}{\partial h} > 0$.

Proof of Proposition 4: Using Equation 4 and taking the derivative with respect to the belief heterogeneity h, we get:

$$\frac{\partial p_x^*}{\partial h} = \frac{2cQ_x}{(c-2h)^2\tau} > 0 \tag{15}$$

Hence we immediately have that the derivative is positive.

Proof of Proposition 5: Using Equation 14 and taking the derivative with respect to the risk tolerance τ, we get:

$$\begin{aligned}\frac{\partial Q_z^*}{\partial \tau} &= \frac{1}{n}\frac{Q_x}{2(c-2h)^2\tau^3} \cdot (c^2\tau + 4h^2\tau - 4h(Q_x + c\tau)) \\ &> 0\end{aligned}$$

The denominator is positive. Furthermore, $4h^2\tau - 4h(Q_x + c\tau) = 4h(h\tau - (Q + c\tau)) > 0$ by the regularity condition.

Next, using Equation 4 and taking the derivative with respect to the risk tolerance τ, we get:

$$\frac{\partial p_x^*}{\partial \tau} = \frac{2hQ}{(2h-c)\tau^2} > 0 \tag{16}$$

since $2h - c > 0$ by the regularity condition.

Proof of Proposition 6: Using Equation 4 and taking the derivative with respect to the quantity of the underlying stock Q_x, we get:

$$\frac{\partial p_x^*}{\partial Q_x} = -\frac{2h}{(2h-c)\tau} < 0 \tag{17}$$

since $2h - c > 0$ by the regularity condition.

Using Equation 14 and taking the derivative with respect to the quantity of the underlying stock Q_x, we get:

$$\frac{\partial Q_z^*}{\partial Q_x} = -\frac{1}{n}\frac{c^2\tau + 4h(h\tau - (Q + c\tau))}{2(2h-c)^2\tau^2} < 0$$

since $2h - c > 0$ and $h\tau - (Q_x + c\tau) > 0$ by the regularity condition.

Proof of Proposition 7: Using Equation 16, I take another derivative with respect to h to get:

$$\frac{\partial p_x^*}{\partial \tau \partial h} = \frac{2cQ_x}{(c-2h)^2\tau} < 0$$

However, the derivative $\frac{\partial p_x^*}{\partial \tau}$ overall is still positive, by Proposition 5,

Then, using Equation 17, I also take another derivative with respect to h to get:

$$\frac{\partial p_x^*}{\partial Q_x \partial h} = \frac{2c}{(c-2h)^2\tau} > 0$$

However, the derivative $\frac{\partial p_x^*}{\partial Q_x}$ overall is still negative, by Proposition 6.

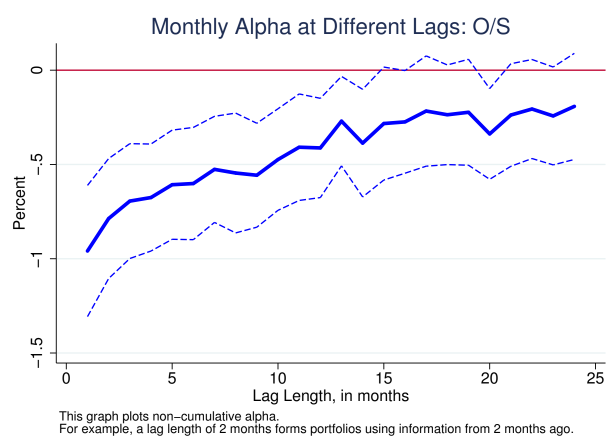
Figure 1: Monthly Alpha at Different Lags

These figures show the effect of changing the lag length of the sorting variable on portfolio alpha, controlling for the Carhart (1997) four-factor model. In Panel (a), the sorting variable is O/S. In Panel (b), the sorting variable is $LevelO/S$. The portfolios are equal weighted monthly returns for the highest quintile less the lowest quintile of the lagged sorting variable. I plot the non-cumulative alpha, i.e. the portfolio holding period is always one month. Formally, suppose I use the quintiles of O/S from j months ago to sort stocks into quintiles. Let $R^{(j)}_{Q5,t+1} - R^{(j)}_{Q1,t+1}$ denote the high-low return spread of this portfolio. I run the regression

$$R^{(j)}_{Q5,t+1} - R^{(j)}_{Q1,t+1} = \alpha^{(j)} + \beta' \cdot f_{t+1} + \epsilon_{t+1}$$

and plot the values of $\alpha^{(j)}$. The dotted lines show the 95% confidence interval, computed using Newey-West standard errors with 12 months of lags. The time span is monthly data from 1996 to 2013 for a total of 211 months.

(a) Using O/S at Different Lags



(b) Using $LevelO/S$ at Different Lags

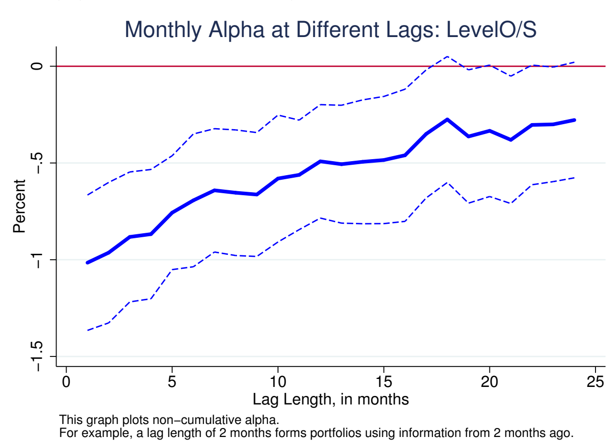


Table 1: List of Key Variables

This table lists the key variables used in this paper.

Variable	**Symbol**	**Description**
Option Stock Volume Ratio	O/S	Options volume (summed over all strike prices, expiration dates, and put/calls) divided by stock volume.
Level Option Stock Ratio	$LevelO/S$	Total options open interest divided by stock shares outstanding. Analogous to O/S, but in levels.
Quintile of Variable X	$Qtile(X)$	Cross-sectional quintile of variable X.
Dispersion of Analyst Forecasts	$Disp$	Standard deviation of analyst forecasts divided by the absolute value of the mean forecast. That is, $\sigma(forecast)/\lvert E[forecast]\rvert$. Prior research has used this variable as a proxy for investor disagreement.
Bid-Ask Spread	$Spread$	(Ask - Bid)/(Midpoint) of the underlying stock.
Stock Return Volatility	Vol	Volatility of stock return over the past 60 trading days.
Simple Return	R	Net simple returns, including dividends.
Excess Return	R^e	Return in excess of the one-month Treasury bill.
Market Cap	$MktCap$	Total equity market capitalization of a firm, using close prices.
CAPM Beta	β	I estimate betas using the Scholes and Williams (1977) method. I compute the betas quarterly and use the estimates from the previous quarter to avoid look-forward bias.
Book/Market Ratio	B/M	(Book Equity)/(Market Capitalization). I lag by two quarters to ensure that the accounting data was already publicly available on each date. Following Fama and French (1993), I define Book Equity = Stockholder's Equity + Deferred Taxes and Investment Tax Credit (if available) - Preferred Stock.
Turnover	$Turnover$	(Share Volume)/(Shares Outstanding)

Table 2: Summary Statistics

This table displays summary statistics of the main variables. Panel (a) displays summary statistics for the full dataset. Panel (b) displays the mean of selected variables by quintile of O/S. Panel (c) displays the mean of selected variables by quintile of $Disp$. O/S is the ratio of option trading volume to stock trading volume. $LevelO/S$ is the level version of O/S; it is the total option open interest divided by stock shares outstanding. $MktCap$ is the market capitalization of the underlying stock in billions of dollars. B/M is the book to market ratio. $Turnover$ is the monthly turnover in the underlying stock. $Spread$ is the bid-ask spread of the underlying stock, expressed as a percent. Vol is the daily return volatility over the last 60 trading days. $Disp$ is the dispersion in analyst forecasts, as defined in Diether, Malloy, and Scherbina (2002). See Table 1 for full list of variable definitions. Data frequency is monthly, spanning 1996 to 2013, and has $N = 300,860$ firm-month observations.

(a) Full Dataset

	mean	sd	p10	p50	p90
O/S, in pct	7.57	14.3	0.44	3.65	18.0
LevelO/S, in pct	48.5	97.7	1.91	16.8	115.7
MktCap, in billions	5.82	19.3	0.28	1.37	11.7
B/M Ratio	0.52	0.46	0.14	0.42	0.98
Monthly Turnover, in pct	21.6	19.8	5.32	15.6	44.7
Spread, in pct	0.61	0.99	0.026	0.16	1.80
Num Firms per Month	1590.1	211.1	1296	1592	1871
Vol, in pct	2.76	1.45	1.32	2.42	4.64

(b) Mean of Selected Variables, by quintile of O/S

	1	2	3	4	5
O/S, in pct	0.47	1.63	3.64	7.70	23.87
LevelO/S, in pct	4.38	11.76	24.44	51.68	146.97
MktCap, in billions	1.75	2.41	3.33	6.05	15.22
B/M Ratio	0.63	0.56	0.52	0.47	0.42
Monthly Turnover, in pct	13.87	17.05	20.60	24.95	30.90
Spread, in pct	0.68	0.65	0.60	0.56	0.55
Vol, in pct	2.45	2.66	2.82	2.92	2.94
Quintile of Disp	2.88	2.92	2.98	3.02	3.13

mean coefficients

(c) Mean of Selected Variables, by quintile of $Disp$

	1	2	3	4	5
StdDev of O/S, in pct	9.23	11.03	12.84	11.72	11.23
StdDev of LvlO/S, in pct	54.32	67.08	84.63	99.81	125.87

mean coefficients

Table 3: Correlation Table of Quintiles of Selected Variables

This table shows the correlation between the quintiles of selected variables. O/S is the ratio of option trading volume to stock trading volume. $LevelO/S$ is the level version of O/S; it is the total option open interest divided by stock shares outstanding. *Disp* is the dispersion in analyst forecasts, as defined in Diether, Malloy, and Scherbina (2002). *Spread* is the bid-ask spread of the underlying stock, expressed as a percent. See Table 1 for full list of variable definitions. Data frequency is monthly, spanning 1996 to 2013, and has $N = 300,860$ firm-month observations.

	Qtile(O/S)	Qtile(LevelO/S)	Qtile(Disp)	Qtile(Spread)
Qtile(O/S)	1			
Qtile(LevelO/S)	0.789***	1		
Qtile(Disp)	0.0603***	0.153***	1	
Qtile(Spread)	-0.155***	-0.163***	0.174***	1

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4: Alpha of a Long-Short Portfolio

This table shows the alphas of long-short portfolios. In Columns (1) - (3), the sorting variable is O/S, the ratio of the total trading volume of options to the trading volume of the underlying stock. In Columns (4) - (6), the sorting variable is $LevelO/S$, the ratio of the total open interest of options to the shares outstanding of the underlying stock. I sort stocks into quintiles based on the previous month's O/S (or $LevelO/S$). The dependent variable is the monthly return spread between the highest quintile and the lowest quintile, $R_{Q5,t+1} - R_{Q1,t+1}$. Portfolios are equal-weighted. Columns (1) and (4) controls for the Carhart (1997) four-factor model. Columns (2) and (5) adds controls for short-term reversal factor (ST Rev) and long-term reversal factor (LT Rev). Columns (3) and (6) adds controls for the Pastor and Stambaugh (2003) liquidity factor (LIQ) and the Frazzini and Pedersen (2014) betting against beta factor (BAB). Time span is monthly data from 1996 to 2013 for a total of 211 months. In parentheses, I display Newey-West t-statistics with lag length 12 months.

$$R_{Q5,t+1} - R_{Q1,t+1} = \alpha + \beta' \cdot f_{t+1} + \epsilon_{t+1}$$

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	O/S	O/S	O/S	LevelO/S	LevelO/S	LevelO/S
Rm-Rf, in %	0.25***	0.24***	0.24***	0.34***	0.33***	0.32***
	(5.7)	(5.3)	(5.0)	(6.6)	(6.1)	(6.0)
SMB, in %	-0.04	-0.01	-0.01	0.17***	0.19***	0.19***
	(-0.6)	(-0.2)	(-0.2)	(3.2)	(2.9)	(2.8)
HML, in %	-0.56***	-0.53***	-0.52***	-0.71***	-0.69***	-0.63***
	(-10.9)	(-7.9)	(-7.1)	(-9.5)	(-7.5)	(-6.2)
UMD, in %	0.08*	0.09*	0.09*	-0.05*	-0.04	-0.03
	(1.8)	(1.7)	(1.7)	(-1.7)	(-1.1)	(-0.7)
ST Rev, in %		0.03	0.03		0.06	0.05
		(0.8)	(0.7)		(1.3)	(1.2)
LT Rev, in %		-0.07	-0.06		-0.06	-0.08
		(-1.0)	(-0.9)		(-0.7)	(-0.9)
LIQ, in %			0.02			0.02
			(0.4)			(0.5)
BAB, in %			-0.01			-0.07
			(-0.2)			(-1.5)
Constant	-0.96***	-0.97***	-0.97***	-1.02***	-1.03***	-1.01***
	(-5.5)	(-5.3)	(-5.3)	(-5.8)	(-5.6)	(-5.3)
Observations	211	211	211	211	211	211
R-squared	0.584	0.588	0.588	0.696	0.699	0.703

t-statistics in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 5: Alpha of a Long-Short Portfolio: O/S, Different Subsamples

This table shows the effect of defining O/S with different subsamples of the options data. Column (1) is O/S defined only using volume of put options. Column (2) only uses volume of call options. Column (3) only uses volume of near-dated options, i.e. expiring within 30 calendar days. Column (4) only uses volume of far-dated options, i.e. expiring after 30 calendar days. For the portfolio sort, I sort stocks into quintiles based on the previous month's O/S (defined using different subsamples of options volume). The dependent variable is the monthly return spread between the highest quintile and the lowest quintile, $R_{Q5,t+1} - R_{Q1,t+1}$. Portfolios are equal-weighted. I control for the return factors of the Carhart (1997) four-factor model. Time span is monthly data from 1996 to 2013 for a total of 211 months. In parentheses, I display Newey-West t-statistics with lag length 12 months.

$$R_{Q5,t+1} - R_{Q1,t+1} = \alpha + \beta' \cdot f_{t+1} + \epsilon_{t+1}$$

VARIABLES	(1) Put	(2) Call	(3) Near	(4) Far
Rm-Rf, in %	0.23***	0.24***	0.29***	0.20***
	(5.9)	(5.4)	(5.7)	(5.7)
SMB, in %	-0.08	-0.04	-0.02	-0.06
	(-1.5)	(-0.6)	(-0.3)	(-1.0)
HML, in %	-0.50***	-0.57***	-0.69***	-0.42***
	(-11.2)	(-9.7)	(-9.1)	(-9.8)
UMD, in %	0.01	0.12***	0.07	0.08*
	(0.3)	(2.7)	(1.5)	(1.7)
Constant	-0.91***	-0.92***	-1.10***	-0.77***
	(-6.2)	(-4.8)	(-5.3)	(-4.8)
Observations	211	211	211	211
R-squared	0.566	0.585	0.659	0.470

t-statistics in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 6: Fama-MacBeth Regression: Cross Section of Individual Stock Returns

This table displays the results of Fama and MacBeth (1973) regressions of excess monthly returns of an individual stock on O/S (and its variants) and on firm characteristics. Excess monthly returns $R^e_{i,t+1}$ are a stock's monthly return less the one month Treasury bill rate. In each regression, I control for the firm characteristics of log market capitalization, log book to market ratio, the CAPM beta, returns over the past 12 months (excluding the most recent month), returns over the past month, and turnover. Column (1) uses the quintile of O/S. Column (2) uses the quintile of $LevelO/S$. Column (3) uses O/S itself. Column (4) uses $LevelO/S$ itself. Data frequency is monthly, spanning 1996 to 2013, and has $N = 300,860$ firm-month observations. In parentheses, I display Newey-West t-statistics with 12 months of lags.

$$R^e_{i,t+1} = b_{0,t} + \lambda \cdot (\text{Measure of } O/S)_{i,t} + b_1 \cdot X_{i,t} + \epsilon_{i,t+1}$$

VARIABLES	(1)	(2)	(3)	(4)
Qtile(O/S)	-0.14***			
	(-3.3)			
Qtile(LevelO/S)		-0.12***		
		(-3.1)		
O/S			-1.77***	
			(-3.2)	
LevelO/S				-0.40***
				(-3.1)
ln(MktCap)	0.02	0.00	0.01	-0.01
	(0.4)	(0.0)	(0.1)	(-0.1)
ln(B/M)	0.14*	0.15*	0.15*	0.16**
	(1.8)	(1.9)	(2.0)	(2.0)
Beta	-0.14	-0.13	-0.15	-0.14
	(-0.9)	(-0.9)	(-1.0)	(-1.0)
$R^i_{-12mo:-1mo}$	-0.00	-0.00	-0.00	-0.00
	(-0.3)	(-0.4)	(-0.4)	(-0.4)
$R^i_{-1mo:-1mo}$	-0.02***	-0.02***	-0.02***	-0.02***
	(-3.1)	(-3.2)	(-3.1)	(-3.2)
Turnover	-0.65	-0.55	-0.77*	-0.46
	(-1.6)	(-1.4)	(-1.7)	(-1.0)
Observations	300,860	300,860	300,860	300,860
R-squared	0.073	0.073	0.072	0.072
Number of months	211	211	211	211

t-statistics in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 7: Interaction Effect with Analyst Forecast Dispersion

Panel (a) shows the average excess returns of a two way portfolio sort between O/S vs analyst forecast dispersion ($Disp$). Following Diether, Malloy, and Scherbina (2002), I define $Disp$ as the standard deviation of analyst forecasts divided by the absolute value of the mean forecast. The right most column shows the average excess return for each quintile of O/S. It is not necessarily the same as the simple average across each row because the quintiles of O/S are not independent of the quintiles of $Disp$. Panel (b) illustrates the same interaction effect using a Fama-MacBeth regression. Unreported controls are log market capitalization, log book to market ratio, the CAPM beta, returns over the past 12 months (excluding the most recent month), returns over the past month, and turnover, all cross-sectionally de-meaned. See Table 1 for full list of variable definitions. Data frequency is monthly, spanning 1996 to 2013, and has 223,049 firm-month observations, after merging with analyst forecast data.

(a) Two Way Portfolio Sort of Average Excess Returns

	Disp Low	2	3	4	*Disp* High	All Stocks
(O/S) Low	0.89	0.84	1.09	0.97	0.94	0.95
2	0.80	0.57	0.35	0.39	0.53	0.54
3	0.57	0.52	0.40	0.14	0.28	0.39
4	0.64	0.28	0.25	-0.22	-0.20	0.14
(O/S) High	0.31	0.30	0.06	-0.03	-0.32	-0.02
(O/S) 5-1	-0.57**	-0.54**	-1.03***	-1.00***	-1.25***	-0.97***
t-statistic	(-2.3)	(-2.0)	(-3.2)	(-2.7)	(-3.4)	(-4.2)

(b) Fama-MacBeth Regression on Individual Stock Returns

$$
\begin{aligned}
R^e_{i,t+1} &= b_{0,t} + \lambda_1 \cdot Qtile(O/S)_{i,t} + \lambda_2 \cdot Qtile(O/S)_{i,t} \times Qtile(Disp)_{i,t} \\
&\quad + \lambda_3 \cdot Qtile(Disp)_{i,t} + b_1 \cdot X_{i,t} + \epsilon_{i,t+1}
\end{aligned}
$$

VARIABLES	(1)	(2)	(3)
Qtile(O/S)	-0.15***	-0.14***	-0.14***
	(-3.7)	(-3.9)	(-3.9)
Qtile(O/S) × Qtile(Disp)			-0.04***
			(-3.2)
Qtile(Disp)		-0.03	-0.03
		(-0.7)	(-0.7)
Observations	223,049	223,049	223,049
R-squared	0.080	0.083	0.084
Number of months	211	211	211
Controls	Y	Y	Y

t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1